CHAPTER TWO
UNITS AND MEASUREMENT

TOPICS COVERED IN THIS CHAPTER

- Introduction
- The international system of units
- Measurement of length
- Measurement of mass
- Measurement of time
- Accuracy, precision of instruments and errors in measurement
- Significant figures
- Dimensions of physical quantities
- Dimensional formulae and dimensional equations
- Dimensional analysis and its applications

1 INTRODUCTION

Quantity:- Anything which can be measured is called quantity.

Phycical Quantity :-A quantity in terms of which law of physics can be expressed and can be measured directly or indirectly is called physical quantity. Types of Physical Quantities:

(i) Fundamental Quantities. (ii) Derived Quantities

(i) Fundamental Quantities:- The quantities which cannot be derived from other quantities called fundamental quantities. - e.g. mass, length and time.

(ii) Derived Quantities:- The quantities which can be derived from fundamental physical quantities are called derived physical quantities. E.g. velocity, acceleration, force etc.

Unit:- Unit of a physical quantity may be defined as the standard of its measurement.

If Q = quantity

u = unit of quantity

Then Q = nu

Where n = no. of times the unit u is contained in Q.

(I) Fundamental Units:- The units of those quantities which can not be derived from other physical quantities are called fundamental units. e.g. units of mass, length, time etc.

(ii) Derived Units:- The units of those quantities which can be derived from other physical quantities are called derived units. e.g. units of velocity, acc., force etc.

Characteristics of a Unit :

A unit should have the following character

1. It should be well defined.
2. It should be a suitable size.
3. It should be easily accessible.
4. It should be easily reproducible.
5. It should not change with time.
6. It should not change with change in physical conditions like temperature, pressure etc.

System of Units :

A complet set of fundamental and derived units for all kinds of the quantities is called *system of units.*

Some common system of Units:

1. F.P S. System: - It is a British Engineering system based upon foot, pound, and second an the fundamental unit of length, mass, and time.

2. C.G.S. System :- It is based on centimeter, gram and second as the fundamental unit of length, mass and time.

3. M.K.S. System: - It is based on meter, kilogram and second as the fundamental Unit of length, mass and time.

2 THE INTERNATIONAL SYSTEM OF UNITS

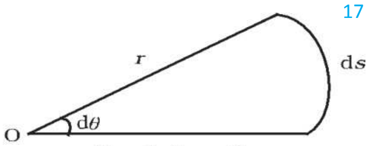
SI System: - It is an International system of unit. S.I. stands for It based upon seven fundamental, two supplementary and a large no. of derived units

SI Base Quantities and Units*

Base quantity	SI Units Name	Symbol	Definition
Length	metre	m	The metre is the length of the path travelled by light in vacuum during a time interval of 1/299,792,458 of a second. (1983)
Mass	kilogram	kg	The kilogram is equal to the mass of the international prototype of the kilogram (a platinum-iridium alloy cylinder) kept at international Bureau of Weights and Measures, at Sevres, near Paris, France. (1889)
Time	second	s	The second is the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom. (1967)
Electric current	ampere	A	The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length. (1948)
Thermo dynamic Temperature	kelvin	K	The kelvin, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water. (1967)
Amount of substance	mole	mol	The mole is the amount of substance of a system, which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon - 12. (1971)
Luminous intensity	candela	cd	The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian. (1979)

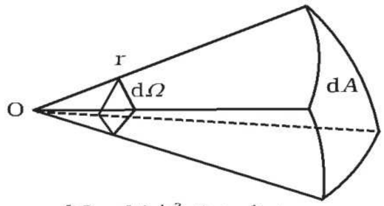
Supplementary Units:

(a) Plane angle dϑ is defined as the ratio of length of arc d*s* to the radius *r* . Fig. (a)



$d\theta = ds/r$ radian

(b) Solid angle dΩ is defined as the ratio of the intercepted area d*A* of the spherical surface, described about the apex O as the centre, to the square of its radius *r*, as shown in fig. (b)



$d\Omega = dA/r^2$ steradian

The unit for plane angle is **radian** with the symbol rad and the unit for the solid angle is **steradian** with the symbol sr. Both these are dimensionless quantities.

Derived Units:

Physical Quantities	**Derived Units**	**Symbol Used**
1. Force	newton	N
2. Work /Energy	jule	J
3. Power	watt	W
4. Electric charge	coulomb	C
5. Electric potential	volt	V
6. electric capacity	farad	F
7. Magnetic flux	weber	Wb, and so on

Some units retained for general use (Though outside SI)

UNITS	**SYMBOL**	**VALUE IN SI UNITS**
minute (time)	min	60s
hour (time)	h	60 min = 3600s
day (time)	d	24h = 86400s
year (time)	y	365.25d = 3.156×10^7s
degree (angle)	°	$1^\circ = \frac{\pi}{180}$ rad
litre (volume)	L	1 $dm^3 = 10^{-3}$ m^3
tone (mass)	t	10^3 kg
carat	c	200 mg
bar (pressure)	bar	0.1 MPa $=10^5$Pa
curie	Ci	3.7×10^{10} s^{-1}
quintal (mass)	q	100 kg
barn (area)	b	10^{-28} m^2
are (area)	a	10^2 m^2
hectare (area)	ha	10^4 m^2
standard atmospheric pressure	atm	101325 Pa = 1.013×10^5 Pa
slug		14.57 kg

ADVANTAGES OF SI

1. **SI is** a **coherent system of units** *i.e.* a system based on a certain set of fundamental units, from which all derived units are obtained by multiplication or division without introducing numerical factors *i.e.* units of a given quantity are related to one another by powers of 10.

2. SI is a rational system of units, as it assigns only one unit to a particular physical quantity.
For example joule is the unit for all types of energy. This is not so in other systems of units. e.g. in MKS system, mechanical energy is in joule, heat energy is in calorie and electric energy is in watt hour.

3. SI is an absolute system of units. There are no gravitational units on the system. The use of factor 'g' is thus eliminated.

4. S.1 is a metric system *i.e.* the multiples and submultiples of units are expressed as powers of 10.

5. In current electricity, the absolute units on the S.I, like *ampere* for current, *volt* for potential difference, *ohm* for resistance, *henry* for inductance, *farad* for capacity arid so on, happen to be the practical units for measurement of these quantities.

SOME MACROSCOPIC UNITS OF DISTANCE:

1. **Astronomical Unit (A.U.) :** *The average distance between the centre of the sun to the centre of the earth is known as one A.U.*

   $$1 \text{ A.U.} = 1.496 \times 10^{11} \approx 1.5 \times 10^{11} \text{ m}$$

2. **Light Year (l.y.) :** *One light year is the distance travelled by light in vacuum in one year.*

   $$1 \text{ l.y.} = 3 \times 10^{8} \times (365 \times 24 \times 60 \times 60)$$
   $$= 9.46 \times 10^{15} \text{ m}$$

3. **Par sec (Parallactic Second):** *One Par sec is the distance at which an arc of 1A.U. long subtends an angle of 1 second.*

   $$1 \text{ Parsec} = 3.084 \times 10^{16} \text{ m}$$

Q. Calculate the relation between A.U. parsec and light year.

Ans. 1 A.U. = 1.5×10^{11} m

1 *l.y* = 9.46×10^{15} m

1 Parsec = 3.1×10^{16} m

So, $\frac{1\ l.y.}{1 A.U.}$ = $(1.5 \times 10^{11}\ m)/(9.46 \times 10^{15}\ m) = 6.3 \times 10^{4}$

Hence, **1 *l.y.* = 6.3×10^{4}**

Also, $\frac{1\ parsec}{1\ l.y.} = \frac{3.1 \times 10^{16}}{9.46 \times 10^{15}} = 3.26$

Hence, **1 parsec = 3.26 *l.y.***

NOTE : 1. Size of universe is nearly 10^{10} l.y.

2. The nearest star, Alpha centaury, outside our solar system is *4.31 l.y.* away from the earth.

SMALLER UNITS OF DISTANCE

1 1 mciron = 1µ or 1µm = 10^{-6}m

2 1 nanometer = 1 nm = 10^{-9}m

3 1 angstrom = 1A° = 10^{-10}m

4 1 fermi = 1 femto metere = 1*fm* = 10^{-15} m

Prefixes for powers of 10

Power of 10	Prefix	Symbol	Power of 10	Prefix	Symbol
10^{-1}	deci	*d*	10^{1}	deca	da
10^{-2}	centi	*c*	10^{2}	hector	*h*
10^{-3}	milli	*m*	10^{3}	kilo	*k*
10^{-6}	micro	µ	10^{6}	mega	M
10^{-9}	nano	*n*	10^{9}	giga	G
10^{-12}	pico	*p*	10^{12}	tera	T
10^{-15}	femto	*f*	10^{15}	peta	P
10^{-18}	atto	*a*	10^{18}	exa	E

Exercise:

I. Is light year a unit of time?
Ans: No, it is unit of distance.
II. Human heart is an inbuilt clock. Comment.
Ans: As human heart beats at a regular rate so it acts as an inbuilt clock.
III. 1 parsec = ----------- light years.
IV. 1 kg = -------------- mg.
V. 1 light year = -------------- metre. Ans: 1.057×10^{-16}
VI. What is difference between A.U. and A^o.
VII. Express a wavelength of 5893A^o in *nm* and *meter*.

ORDER OF MAGNITUDE

The power of 10 nearest to the actual value of the magnitude of the quantity is called order of magnitude

e.g. Order of magnitude of size of atomic nucleus = 10^{-14} m
Order of magnitude of height of a person = 10^0m
Order of magnitude of radius of earth = 10^7m.

MEASUREMENT OF LENGTH

Length can be measured by (1) Direct Methods or by (2) Indirect Method.

Direct Methods:

These methods involves the use of

i. *metre scale* (10^{-3} m to 10^2m)
ii. vernier calipers (up to 10^{-4}m)
iii. screw gauge (up to 10^{-5}m)

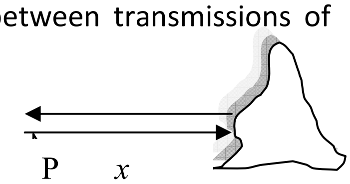
Indirect Method:

Beyond the range 10^{-5} to 10^2 we use indirect methods.

I. REFLECTION METHODS

i. **Echo Method:** *It is method used to find the distance (x) of a hill from a given point.*

In this method sound wave sent from the point of observation P to hill. The sound wave is reflected back by hill. By finding the time difference (t) between transmissions of sound and heard of echo.



The distance travelled by sound wave = $x + x = v \times t$

$$\Rightarrow 2x = v \times t$$

$$\Rightarrow x = \frac{v \times t}{2}$$

Hence we can find x using $x = \frac{v \times t}{2}$, *where v* is velocity of sound.

ii. **Laser Method:** *It is used to find the distance of moon from earth.*
The laser beam is transmitted from earth is received back on earth after reflection from moon.
The time interval (t) between transmission and reception of beam is measured.

If c = velocity of Laser

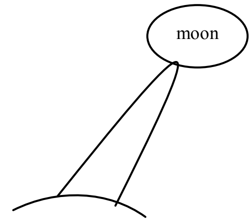
Then distance = velocity × time

$x + x = c \times t$

$2x = c \times t$

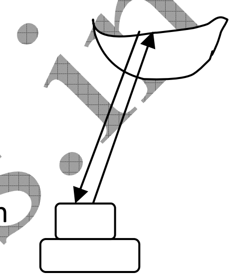
Or $x = \frac{c \times t}{2}$

Where x = distance between earth and moon.



iii. **Sonar Method: (Sound Navigations and Ranging):** *This method is used to find the depth of ocean and to locate submarine inside the water.*

In this method ultrasonic waves are sent through ocean from transmitter to submarine. These ultrasonic wav are reflected from the submarine. By finding the time difference (t) between transmitted and reflected waves, the distance x of submarine can be find using the formula



$$x = \frac{\boldsymbol{c \times t}}{\boldsymbol{2}}$$

II. TRIANGULAR METHOD: (a) To determine the height of an accessible object like tree or pole:

Let we have to find out the height, h =AB

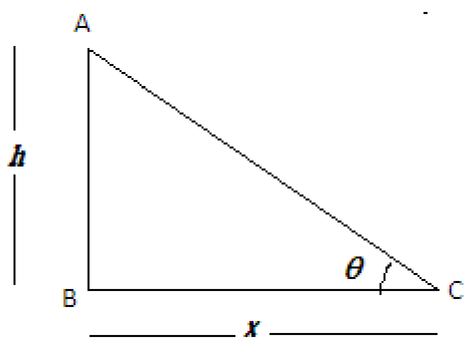
Let c is the point of observation, as shown in figure.

θ = ∠ACB = angle of elevation, it can be measured by sextant.

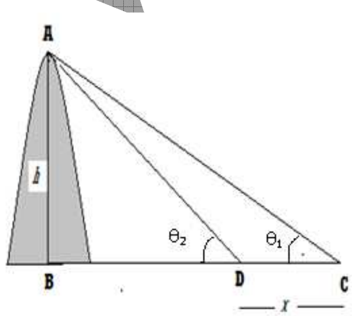
Then in Δ ABC

$$\frac{AB}{BC} = \frac{h}{x} = \tan\theta$$

Or $\boldsymbol{h = x \tan\theta}$



Hence by measuring x and θ we can find out height h of any tree or pole.



(b) To determine the height of an inaccessible object like a mountain or a tree on other side a river.

To determine the height AB = h

To find it, we measure two angle of elevation θ_1 and θ_2 from two points C and D respectively.

Let CD = x

Then in Δ ABC, cot $\theta_1 = \frac{CB}{AB}$

in Δ ABD, $\cot\theta_2 = \frac{DB}{AB}$

$\cot\theta_1 - \cot\theta_2 = \frac{CB}{AB} - \frac{DB}{AB} = \frac{CB-DB}{AB} = \frac{x}{h}$

$$h = \frac{x}{\cot\theta 1 - \cot\theta 2}$$

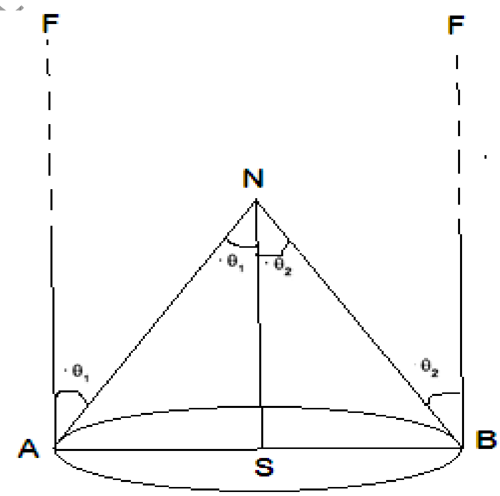
hence by knowing θ_1 , θ_2 and *x* we can calculate *h*.

Parallax Method:

This method is used to determine distance of stars which are less than 100 light years away .

Let we have to find out the distance (*d*) of a nearby star N. let A and B are the two positions of earth revolving around the sun. The position B is diametrically opposite to position A.

Let F is any star at very large distance from the earth such that its direction and position w.r.t. earth remains constant.



Now $\angle FAN = \angle ANS = \theta_1$ (Alternate angles)

& $\angle FBN = \angle BNS = \theta_2$ (Alternate angles)

Therefore, $\angle ANB = \angle ANS + \angle BNS$

$= \theta_1 + \theta_2$

As, $angle = \frac{arc}{radius}$

$\Rightarrow \theta_1 + \theta_2 = \frac{AB}{AN}$

$AN = \frac{AB}{\theta_1 + \theta_2}$

Now $AB = 2AS = 2AU$

$\therefore \quad AN = d = \frac{2AU}{\theta_1 + \theta_2}$

Hence by knowing θ_1 and θ_2 we can calculate AN = BN = *d*.

or $\theta = \frac{D}{r}$

=> $D = \theta . r$

Thus diameter D i.e. size of planet (or moon) can be calculated by knowing & r.

Size of Moon or Planet:

Let *r* = distance of planet or moon from the earth.

θ = angle subtended by diameter AB of planet or moon at any point on the earth.

As, angle = $\frac{arc}{radius}$

$\Longrightarrow \theta = \frac{AB}{r}$

$\Longrightarrow \theta = \frac{d}{r}$ or $\boldsymbol{d = \theta \times r}$

Hence, the diameter *'d'* can be calculated by measuring 'θ' and using '*r*'

Numerical: Find the value of following in radians.(i) 1^o (ii) 1' (iii) 1''

(i) $1^o = \frac{\pi}{180}$ radian = 1.745×10^{-2} radian

(ii) $1' = \frac{1degree}{60}$ = $1.745\times 10^{-2}/60$ radian $\cong 2.91 \times 10^{-4}$ radian

(iii) $1'' = \frac{1minute}{60}$ = $2.91 \times 10^{-4}/60$ radian $\cong 4.85 \times 10^{-6}$ radian

Numerical: if a satellite is observed from two diametrically opposite points P and Q on its planet. The angle α subtended at the satellite by the two directions of observation is $1^o54'$. Calculate the distance of satellite from the planet, if diameter of planet = 1.276×10^7m.

Solution

$\alpha = 1^o54' = 114' = 3.32 \times 10^{-2}$rad

Also, if diameter of planet, D = PQ = 1.276×10^7m.

Hence distance = $\frac{diameter}{angle}$ = $1.276\times10^7/3.32 \times 10^{-2}$ = 3.84×10^8 m

ERROR: The difference between true value and measured value is known as error in measurement.

Accuracy: The accuracy of a measurement is a measure of how close the measured value is to the true value of the quantity.

Precision: precision tells us to what limit the quantity is measured.

Types of Errors

Errors may be divided into following three types

(I) systematic errors and

(II) Random errors.

(III) Gross errors

(I) Systematic errors

The **systematic errors** are those errors that tend to be in one direction, either positive or negative.

Some of the sources of systematic errors are :

(a) **Instrumental errors** that arise from the errors due to imperfect design or zero error in the instrument, etc.
For example, in a vernier callipers the zero mark of vernier scale may not coincide with the zero mark of the main scale, or simply an ordinary metre scale may be worn off at one end.

(b) Imperfection in experimental technique or procedure
To determine the temperature of a human body, a thermometer placed under the armpit will always give a temperature lower than the actual value of the body temperature.

(c) **Personal errors** that arise due to an individual's bias, lack of proper setting of the apparatus or individual's carelessness in taking observations without observing proper precautions, etc.
For example, if people, by habit, always hold your head a bit too far to the right while reading the position of a needle on the scale, you will introduce an error due to **parallax**.

NOTE: Systematic errors can be minimized by improving experimental techniques, selecting better instruments and removing personal bias as far as possible. For a given set-up, these errors may be estimated to a certain extent and the necessary corrections may be applied to the readings.

(II) Random errors or Chance Errors

The **random errors** are those errors, which occur irregularly and hence are random with respect to sign and size. These can arise due to random and unpredictable fluctuations in experimental conditions (temperature, voltage supply etc).
For example, when the same person repeats the same observation, he may get different readings every time.

(III) Gross Error:

The errors due to carelessness of the observer are known as Gross Errors. e.g. Recording the observations wrongly, using the wrong values of observations in calculations.

Least Count: The smallest value that can be measured by the measuring instrument is called its **least count**. All the readings or measured values are good only up to this value.

Least count error The **least count error** is the error associated with the resolution of the instrument. For example, a vernier callipers has the least count as 0.01 cm; a spherometer may have a least count of 0.001 cm.
Least count error belongs to the category of random errors but within a limited size; it occurs with both systematic and random errors.
Using instruments of higher precision etc., we can reduce the least count error.

Main Point: Repeating the observations several times and taking the arithmetic mean of all the observations, the mean value would be very close to the true value of the measured quantity.

Absolute Error, Relative Error and Percentage Error
Absolute Error: The magnitude of the difference between the true value of the quantity and the individual measurement value is called the absolute error of the measurement. This is denoted by $|\Delta a|$.
If true value is not known then we considered arithmetic mean as the true value. Then the errors in the individual measurement values are

$\Delta a_1 = a_{mean} - a_1$,
$\Delta a_2 = a_{mean} - a_2$,
....
....
$\Delta a_n = a_{mean} - a_n$

Mean absolute error: The arithmetic mean of all the *absolute errors* is taken as the *final* or *mean absolute error* of the value of the physical quantity *a*. It is represented by Δa_{mean}.
Thus,

$$\Delta a_{mean} = (|\Delta a_1|+|\Delta a_2|+|\Delta a_3|+...+|\Delta a_n|)/n$$

Relative Errors: *The relative error is the ratio of the mean absolute error* Δa_{mean} *to the mean value* a_{mean} *of the quantity measured.*

$$\text{Relative error} = \Delta a_{mean}/a_{mean}$$

Percentage Error: *When the relative error is expressed in per cent, it is called the percentage error (δa).*
Thus, Percentage error $\delta a = (\Delta a_{mean}/a_{mean}) \times 100\%$

Combination of Errors

a) Error of a sum or a difference

Suppose Z = A + B. --------------(i)

Let ΔA = absolute error in measurement of quantity A

ΔB = absolute error in measurement of quantity B

 ΔZ = absolute error in sum Z of A and B.

Then $A \pm \Delta A$ = measured value of A

$B \pm \Delta B$ = measured value of B.

$Z \pm \Delta Z$ = measured value of sum Z of A and B.

So, (i) becomes

$Z \pm \Delta Z = (A \pm \Delta A) + (B \pm \Delta B)$

$Z \pm \Delta Z = (A + B) \pm (\Delta A + \Delta B)$

Or $\Delta Z = \pm \Delta A \pm \Delta B$

So possible errors in Z are $+\Delta A + \Delta B$, $-\Delta A + \Delta B$, $+\Delta A - \Delta B$, $-\Delta A - \Delta B$.

& maximum possible error in $Z = \pm (\Delta A + \Delta B)$

Similarly, for the difference $Z = A - B$, we have

$Z \pm \Delta Z = (A \pm \Delta A) - (B \pm \Delta B)$

$= (A - B) \pm \Delta A \pm \Delta B$

or, $\pm \Delta Z = \pm \Delta A \pm \Delta B$

The maximum value of the error ΔZ is again $Z = \pm (\Delta A + \Delta B)$

Hence the rule : **When two quantities are added or subtracted, the absolute error in the final result is the sum of the absolute errors in the individual quantities.**

(b) Error of a product or a quotient

Suppose $Z = A \times B$. -------------- (i)

Let ΔA = absolute error in measurement of quantity A

ΔB = absolute error in measurement of quantity B

 ΔZ = absolute error in product Z of A and B.

Then $A \pm \Delta A$ = measured value of A

$B \pm \Delta B$ = measured value of B.

$Z \pm \Delta Z$ = measured value of product Z of A and B.

$Z \pm \Delta Z = (A \pm \Delta A)(B \pm \Delta B)$

$= AB \pm B\,\Delta A \pm A\,\Delta B \pm \Delta A\,\Delta B$.

Dividing LHS by Z and RHS by AB we have,

$1 \pm \frac{\Delta Z}{Z} = 1 \pm \frac{\Delta A}{A} \pm \frac{\Delta B}{B} \pm \frac{\Delta A}{A}\frac{\Delta B}{B}$.

Since ΔA and ΔB are small, we shall ignore their product.

Hence the maximum relative error

$\Delta Z/ Z = (\Delta A/A) + (\Delta B/B)$.

Similarly, for division we also have $\Delta Z/ Z = (\Delta A/A) + (\Delta B/B$

Hence the rule: When two quantities are multiplied or divided, the relative error in the result is the sum of the relative errors in the multipliers.

(c) Error due to power of a quantity.

Suppose $Z = A^2$,

Then,

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta A}{A} = 2\,\frac{\Delta A}{A}$$

Hence, the relative error in A^2 is two times the error in A.

In general, if $Z = A^p\, B^q / C^r$

Then,

$$\frac{\Delta Z}{Z} = p\,\frac{\Delta A}{A} + q\,\frac{\Delta B}{B} + r\,\frac{\Delta C}{C}$$

.

Hence the rule: The relative error in a physical quantity raised to the power *k* is the *k* times the relative error in the individual quantity.

SIGNIFICANT FIGURES

Significant figures in the measured value of a physical quantity gives the number of digits in which we have confidence**.**

Significant figures may be defined as the reliable digits plus the first uncertain digit are known as significant digits or significant figures.

Larger the number of significant figures obtained in a measurement, greater is the accuracy of the measurement and vice-versa.

RULES FOR COUNTING THE SIGNIFICANT FIGURES:

1. **All the non-zero digits are significant.** e.g 1254 has *four* significant figures

2. **All the zeros between two non-zero digits are significant, no matter where the decimal point is, if at all.** e.g 1004 has *four* significant figures

3. **If the number is less than 1, the zero(s) on the right of decimal point but to the left of the first non-zero digit are not significant.** [In $\underline{0.00}$ 2308, the underlined zeroes are not significant].

4. **The terminal or trailing zero(s) in a number without a decimal point are not significant.** [Thus 145 m = 14500 cm = 145000 mm has *three* significant figures, the trailing zero(s) being not significant

5. **The trailing zero(s) in a number with a decimal point are significant.** [The numbers 8.500 or 0.007900 have four significant figures each.]

OPERATIONS ON SIGNIFICANT FIGURES

Addition or Subtraction: In addition or subtraction, the final result should retain as many decimal places as are there in the number with the least decimal places

e.g. 10.342 + 3.12 = 13.46

(Not 13.462, because result should retain two decimal places as there are least two decimal places in 3.12)

13.682 – 13.6 = 0.1 (on rounding off)

Multiplication or division: In multiplication or division, the final result should retain as many significant figures as are there in the original number with the least significant figures.

e.g. $3.145 \times 3.01 = 9.47$ (not 9.46645, as result can have at the most three sig. figures)

& $8500 \div 9.42 = 910$ (on rounding off, not 902.33546, as result can have at the most two sig. figures)

Rounding off the Uncertain Digits

1. **Preceding digit is raised by 1 if the insignificant digit to be dropped (the underlined digit in this case) is more than 5, and is left unchanged if the latter is less than 5**.

   4.356 = 4.36 (on rounding off up to three significant digits)

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2. But if the insignificant digit is 5 then if the preceding digit is even, the insignificant digit is simply dropped and, if it is odd, the preceding digit is raised by 1.

   5.3245 = 5.324 (on rounding off up to four significant digits)

   5.3275 = 5.328 (on rounding off up to four significant digits)

EXERCISE: 1.State the number of significant figures in the following:

(a) 0.006 m^2

(b) 3.64 × 1024 kg

(c) 0.2360 g cm^{-3}

(d) 5.320 J

(e) 9.032 N m^{-2}

(f) 0.0007042 m^2

Sol: (a) 1 (b) 3 (c) 4 (d) 4 (e) 4 (f) 4

2. The length, breadth and thickness of a rectangular sheet of metal are 4.234 m, 1.005 m, and 2.01 cm respectively. Give the area and volume of the sheet to correct significant figures.

Sol: Area = (4.234 × 1.005) × 2 = 8.51034= 8.51 m^2

Volume = 4.234 ×1.05 × (2.01 × 10^{-2}) = 8.55289 × 10^{-2}= 0.0855 m^{-3}

3. The mass of a box measured by a grocer's balance is 2.300 kg. Two gold pieces of masses 20.15 g and 20.17 g are added to the box. What is (a) the total mass of the box, (b) the difference in the masses of the pieces to correct significant figures ?

Ans: (a) 2.3 kg (b) 0.02 kg

4. A physical quantity *P* is related to four observables *a, b, c* and *d* as follows :

$P = a^3b^2/(\sqrt{c}\,\mathbf{d})$

The percentage errors of measurement in *a*, *b*, *c* and d are 1%, 3%, 4% and 2%, respectively. What is the percentage error in the quantity *P* ? If the value of *P* calculated using the above relation turns out to be 3.763, to what value should you round off the result ?

Ans: % age error in P is = 13%.

DIMENSIONS

We may define *the dimensions of a physical quantity as the powers to which the fundamental units of mass, length and time have to be raised to represent a derived unit of the quantity.*

For example, $\text{velocity} = \frac{\text{displacement}}{\text{time}} = \frac{[L]}{[T]}$

Or velocity = $[L^1 T^{-1}] = [M^0 L^1 T^{-1}]$

Hence the dimensions of velocity are: *zero* in mass, *+1* in length and *-1* in time.

DIMENSIONAL FORMULA

The expression which shows how and which of the base quantities represent the dimensions of a physical quantity is called the dimensional formula of the given physical quantity.
For example, the dimensional formula of the volume is [M° L^3 T°],
and dimensional formula of speed or velocity is [M° L T-1].

Similarly, [M° L T^{-2}] is the dimensional formula of acceleration and [M L^{-3} T°] that of mass density.

DIMENSIONAL EQUATION:

An equation obtained by equating a physical quantity with its dimensional formula is called the dimensional equation of the physical quantity.

For example, the dimensional equations of volume [*V*], speed [*v*], force [F] and mass density [ρ] may be expressed as

$[V] = [M^0 L^3 T^0]$
$[v] = [M^0 L T^{-1}]$
$[F] = [M L T^{-2}]$
$[\rho] = [M L^{-3} T^0]$

Dimensional Formula for Some Frequently Used Physical Quantities:

S. No.	Name of Physical Quantities	Formula	Dimensional Formula
1	Area	Length × Breadth	$L \times L = L^2 = [M^0L^2T^0]$
2	Volume	l×b×h	$L \times L \times L = L^3 = [M^0L^3T^0]$
3	Density	$\frac{mass}{volume}$	$\frac{M}{L^3} = [M^1L^{-3}T^0]$
4	Velocity or Speed	$\frac{\text{Distance travelled}}{\text{Time taken}}$	$\frac{L}{T} = [M^0L^1T^{-1}]$
5	Momentum	Mass × velocity	$M \times [M^0L^1T^{-1}] = [M^1L^1T^{-1}]$or
6	Acceleration	$\frac{\text{Change in velocity}}{\text{Time taken}}$	$\frac{L^1T^{-1}}{T} = [M^0L^1T^{-2}]$
7	Force	Mass × Accleration	$M\times [M^0L^1T^{-2}] = [M^1L^1T^{-2}] = [MLT^{-2}]$
8	Work	Force × Displecement	$[M^0L^1T^{-2}] \times [L] = [M^1L^2T^{-2}]$
9	Pressure	$\frac{Force}{Area}$	$\frac{[M^1L^1T^{-2}]}{[L^2]} = [M^1L^{-1}T^{-2}]$
10	Impulse	force× time	$[M^1L^1T^{-2}]\times[T] = [M^1L^1T^{-1}]$
11	Energy	$\frac{1}{2}mv^2$, mgh	$[M^1L^2T^{-2}]$ or $[ML^2T^{-2}]$

12	Power	$\frac{\text{work or Energy}}{\text{time}}$	$\frac{[ML^2T^{-2}]}{T}$ = $[ML^2T^{-3}]$
13	Angle	$\frac{\text{Arc}}{\text{Radius}}$	$\frac{L}{L}$ =$[M^0L^0T^0]$= No Dimensions
14	Stress	$\frac{\text{Force}}{\text{Area}}$	$\frac{[M^1L^1T^{-2}]}{[L^2]}$ = $[M^1L^{-1}T^{-2}]$
15	Strain	$\frac{\text{Change in configuration}}{\text{original configuration}}$	No dimensions
16	Coefficient of elasticity	$\frac{\text{Stress}}{\text{Strain}}$	$[M^1L^{-1}T^{-2}]$ or $[ML^{-1}T^{-2}]$
17	Surface tension	Force per unit length	$[ML^0T^{-2}]$
18	Coefficient of viscosity	$\frac{\text{force} \times \text{distance}}{\text{area} \times \text{velocity}}$	$\frac{[M^1L^1T^{-2}]\times L}{[L^2]\times[L^1T^{-1}]}$ = $[M^1L^{-1}T^{-1}]$
19	Gravitational Constant (G)	G =$\frac{\text{Force}\times\text{distance}^2}{\text{mass}\times\text{mass}}$	$[M^{-1}L^3T^{-2}]$
20	Moment of Inertia	$\text{mass}\times(\text{distance})^2$	$[ML^2T^0]$
21	Radius of Gyration	Distance	[L]
22	Angular Displacement	Angle	No Dimensions
23	Angular Velocity	$\frac{\text{angular displacement}}{\text{time taken}}$	$\frac{1}{T}$ = $[M^0L^0T^{-1}]$
24	Angular Acceleration	$\frac{\text{angular velocity}}{\text{time taken}}$	$\frac{1}{T^2}$ = $[M^0L^0T^{-2}]$
25	Angular Momentum	MI× angular velocity	$[M^1L^2T^{-1}]$
26	Torque	MI × angular acc.	$[M^1L^2T^{-2}]$
27	Force constant	Force/length	$[ML^0T^{-2}]$
28	Specific Heat	$\frac{\text{Heat}}{\text{mass} \times \text{temperature}}$	$[M^0L^2T^{-2}K^{-1}]$
29	Latent Heat	$\frac{\text{Heat}}{\text{mass}}$	$[M^0L^2T^{-2}]$
30	Frequency	No.of vibrations per second	$\frac{1}{T}$ = $[M^0L^0T^{-1}]$
31	Planck's Constant(h)	$\frac{\text{Energy}}{\text{Frequency}}$	$\frac{M^1L^2T^{-2}]}{[M^0L^0T^{-1}]}$ = $[M^1L^2T^{-1}]$

Application of Dimensional Analysis:

1. To check the correctness or consistency of a given formula
2. To derive relationship among various physical quantities.
3. To convert one system of unit into other system.

1. To check the correctness or consistency of a given formula

The correctness or consistency or accuracy of a formula can be checked by applying the **principle of homogeneity of dimensions.** *According to this principle dimensions of each term on the both sides of the formula is always same.*

Example: to check the accuracy of $v^2 - u^2 = 2as$

Solution:

The dimensions of LHS = $[LT^{-1}]^2 - [LT^{-1}]^2 = [L^2 T^{-2}] - [L^2 T^{-2}] = [L^2 T^{-2}]$

The dimensions of RHS = 2aS = $[L T^{-2}]$ $[L]$ = $[L^2 T^{-2}]$

The dimensions of LHS and RHS are the same and hence the equation is dimensionally correct.

Note: length + length = length

Exercise: A book with many printing errors contains four different formulas for the displacement *y* of a particle undergoing a certain periodic motion :

(a) $y = a \sin 2\pi\ t/T$

(b) $y = a \sin vt$

(c) $y = (a/T) \sin t/a$

(d) y=(a 2) $(\sin 2\pi\ t/T + \cos 2\pi\ t/T)$

(a = maximum displacement of the particle, v = speed of the particle. T = time-period of motion). Rule out the wrong formulas on dimensional grounds.

2. **To derive relationship among various physical quantities.**

   Let a physical quantity Q depends upon the quantities q_1 , q_2, and q_3 such that:

   $$Q \propto q_1^a q_2^b q_3^c$$

   Or $$Q = k\, q_1^a q_2^b q_3^c \leftarrow (1)$$

Where k is dimensionless constant.

Now, we will apply the following steps -

i) Write dimensional formula of each quantity on both sides of equation (1)

ii) Equate the powers of M, L and T and solve three equations so obtained and find a, b and c.

iii) Now we will put the values of a, b and c in equation (1), hence we get a new physical relation(formula)

Example: Consider a simple pendulum, having a bob attached to a string, which oscillates under the action of the force of gravity. Suppose that the period of oscillation of the simple pendulum depends on its length (l), mass of the bob (m) and acceleration due to gravity (g). Derive the expression for its time period using method of dimensions.

Solution: The dependence of time period T on the quantities l, g and m as a product may be written as :

$$T = k\, l^a g^b m^c \qquad \leftarrow (1)$$

Where k is dimensionless constant and *a,b and* are the exponents.

By considering dimensions on both sides, we have $[M^0 L^0 T^1] = [L]^a\ [L^1 T^{-2}]^b [M]^c$

$$= [M^c L^{a+b} T^{-2b}]$$

On equating the dimensions on both sides, we have

$a + b = 0$; $-2b = 1$; and $c = 0$

So that $b = -\frac{1}{2}$ and $a = \frac{1}{2}$

Then, from equation (1) we have $T = k\, l^{\frac{1}{2}} g^{-\frac{1}{2}} m^0$

i.e. $$T = k\sqrt{l/g}$$

3. To convert one system of unit into other system.

Let M_1 ,L_1 and T_1 are fundamental units of mass, length and time in one system of unit.
& M_2 ,L_2 and T_2 are fundamental units of mass, length and time in another system.
Let a, b and c be the dimensions of the given quantity in mass, length and time respectively.
If n_1 is the numerical value of the quantity in one system then that n_2 in other system is given by the formula:

$$n_2 = n_1\left(\frac{M_1}{M_2}\right)^a \left(\frac{L_1}{L_2}\right)^b \left(\frac{T_1}{T_2}\right)^c$$

Example: Convert 100 joules into eargs
Solution: Quantity = Work
Dimensional Formula of Work = $[M^1L^2T^{-2}]$
Here *a = 1, b = 2* and *c = -2*

Given system (SI)	To convert in (CGS)
M_1 = 1 kilogram	M_1 = 1 gram
L_1 = 1 metre	L_1 = 1 cm
T_1 = 1 second	T_1 = 1second
n_1 = 100	n_1 = ?

then using

$$n_2 = n_1\left(\frac{M_1}{M_2}\right)^a \left(\frac{L_1}{L_2}\right)^b \left(\frac{T_1}{T_2}\right)^c$$

we have

$$n_2 = 100\left(\frac{1\text{kilogram}}{1\text{gram}}\right)^1 \left(\frac{1\text{ metre}}{1\text{cm}}\right)^2 \left(\frac{1\text{second}}{1\text{second}}\right)^{-2}$$
$$= 100 \times 1000 \times (100)^2 \times 1$$
$$= 10^9$$

Hence, 100 jule = 10^9erg

Limitations of Dimensional Analysis:

1. Using dimensional analysis we cannot find the value of dimensionless constant.
2. We cannot derive the relation containing exponential and trigonometric functions.
3. It cannot inform that whether a quantity is scalar or vector.
4. It cannot find the exact nature of plus or minus, connecting two or more terms in formula.
5. The relation containing more than three physical quantities cannot be derived using dimensional analysis.